

SYNCHRONIZATION METHODS FOR COMMUNICATION WITH CHAOS OVER BAND-LIMITED CHANNELS

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SUMMARY

Methods of chaos synchronization applied to communication with chaotic signals over band-limited channels are studied numerically and experimentally. The first group of methods employs a filter in the receiver which is designed to compensate channel distortions of the transmitted signal. The second approach does not use the compensation but relies on the design of chaotic oscillators which shapes the spectrum of chaotic waveforms sent through the channel. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: chaos; synchronization; communication

1. INTRODUCTION

Recent interest in chaos synchronization is supported, in part, by the hope to build a common framework for the design of self-synchronizing analog, digital, and hybrid (analog–digital) chaotic scramblers which can be used for private communication. Several methods of scrambling of an information signal with analog chaotic signal at the transmitter and extracting the message from chaos at the receiver have been proposed. In one class of these methods, the information signal $m(t)$ is added to the chaotic output $x_e(t)$ generated by a chaotic oscillator whose dynamics do not depend on $m(t)$. The mixture $x_e(t) + m(t)$ is transmitted to the receiver where it is used as a driving signal for response system with matched parameters. Various implementations of the response systems were proposed, see for example References 1–6 and references therein. The common shortcoming of such methods of communication is that the driving signal which is ‘distorted’ by the message $m(t)$, does not perfectly fit the response system. As a result, recovered message $m_r(t)$ will always contain some traces of chaotic waveforms no matter how perfectly the parameters of the decoder (response system) match those of the encoder (driving system). The amplitude of the chaotic contamination is small only if $m(t)$ is small as compared to the chaotic signal $x(t)$.

A different approach to the problem of chaotic encoding and decoding was suggested in a number of papers.^{7–13} The main idea of this approach is that information signal $m(t)$ is injected into the feedback loop of the chaotic encoder. As a result the distorted chaotic signal $x_e(t) + m(t)$ returns back to the generator and influences the dynamics of the encoder. Now when the signal $x_e(t) + m(t)$ is applied to the decoder, it generates oscillations of the response system which are identical to the oscillations in the encoding generator. The message $m(t)$ can be recovered using the open feedback loop of the response system. In this case (in the absence of channel distortions), after initial transients, the information can be restored exactly. Since in this

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Contract/grant sponsor: US Department of Energy; contract/grant number: DE-FG03-95ER14516

Contract/grant sponsor: US Army Research Office; contract/grant number: DAAG55-98-1-0269

approach the information signal directly affects the dynamics of the encoder as its time-varying parameter, we will call it chaos modulation.

The fact that the chaotic signal is broadband, poses serious problems if it is to be transmitted through a band-limited channel. Due to the non-linear nature of the response system, if some part of the spectral components of the chaotic driving signal is not transmitted, or distorted, it affects the whole spectral domain of the signals produced in the response system. Even simple variations of channel gain or pure phase distortions in the channel (the so-called all-pass filters) may spoil drastically the response behaviour.¹⁴ Such high susceptibility of response behaviour to even small distortions of spectral components of the received chaotic signal is the major obstacle for utilization of analog chaotic scramblers in wireless communication devices, see for example Reference 15.

Several recent papers^{14,16-19} addressed this problem by attempting to eliminate the distortion imposed by the channel either by using an inverse filter^{14,16} or by augmenting the distorted signal by the signal of the second system passed through a complimentary band-stop filter¹⁷ (see Figure 1). For a mild first-order filter simulating the channel distortion, an exact synchronization of chaotic oscillations was achieved, however with less degree of stability as compared with the no-filter case. Both of these methods assume that the channel impulse response function is known at the receiver. In practice, however, channel characteristics are often unknown *a priori* and may change in the course of the transmission. Moreover, the stable inverse filter required for the first implementation (Figure 1(a)) is available only in very limited cases. In the following, we will only discuss systems which do not require inverse filtering.

In this paper, we consider possible ways to modify self-synchronizing analog chaotic scramblers in order to maintain private communication over band-limited channels. In Section 2 we discuss information transmission using channel compensation. In Section 3 we describe two new alternative ways to cope with channel distortions which both rely on the design of chaotic systems such that only a narrow-band signal is being sent through a channel. We propose to communicate by means of a chaotic carrier which is wide enough

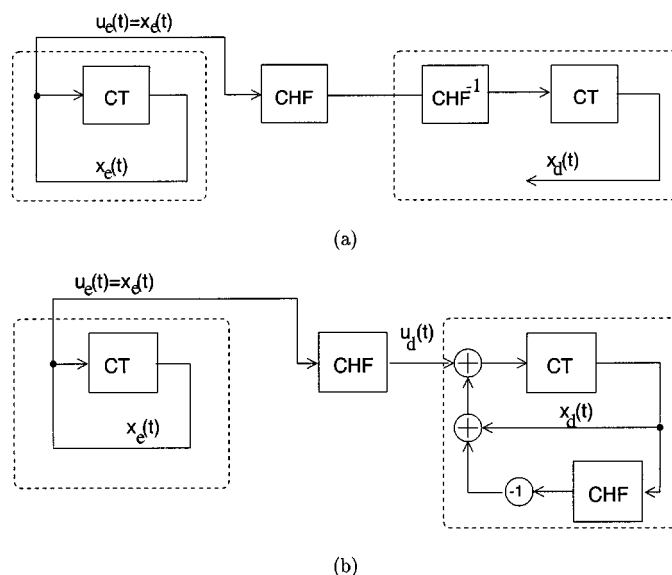


Figure 1. (a) Block-diagram of channel equalization method based on filter inversion;¹⁶ (b) block-diagram of channel equalization method based on complimentary filtering in the feedback loop of the response system.¹⁷ Chaos generation is provided by closed loop dynamics of the chaotic transformer (CT). CHF is the channel filter. CHF⁻¹ is the inverse channel filter. Blocks ⊕ are adders, and block ‘-1’ is an inverter

(spectrally) to hide the information signal, but is still narrow enough to fit into the channel bandwidth without much distortion. In Section 4 some results of experiments with electronic circuits are presented.

2. COMMUNICATION WITH CHAOTIC SIGNALS—CHANNEL COMPENSATION

In this section we consider information masking using chaos modulation in the presence of channel distortions. The original method⁷ of signal transmission with chaos is illustrated by Figure 2(a). The message $m(t)$ is injected into the feedback loop of the driving system and is recovered at the decoder by subtracting the output of the chaotic transformer (CT) $x_d(t)$ from the received signal $u_d(t) = u_e(t)$. Without channel distortions this method yields an exact recovery of the message after an initial transient. Let us apply this method to the scheme of Figure 1(b) (see Figure 2(b)). At the response system, the message is recovered by

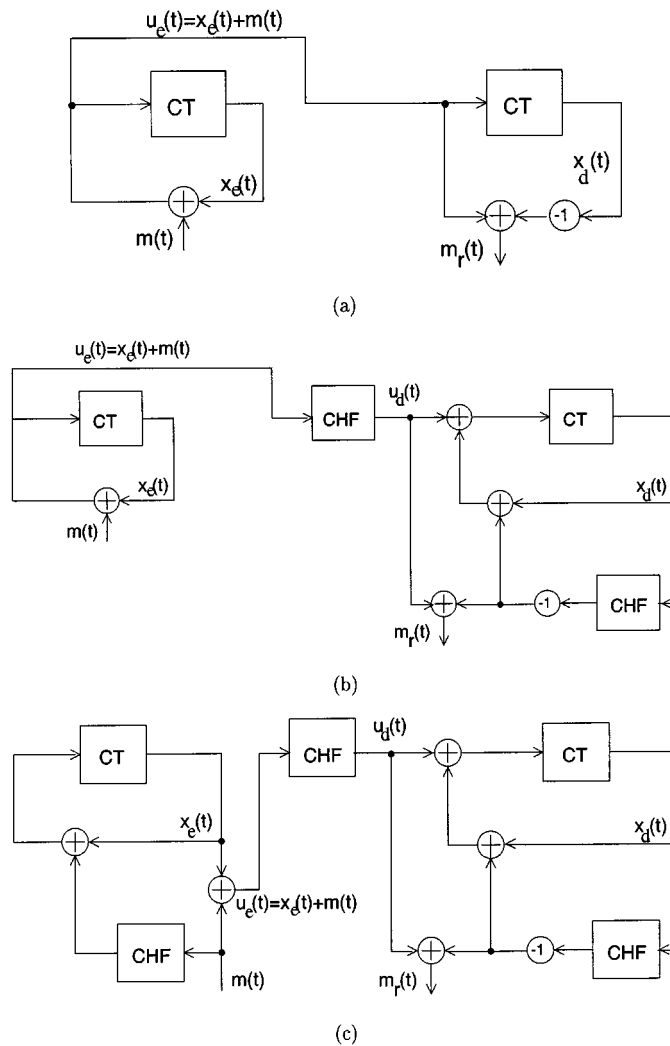


Figure 2. (a) Block-diagram of the standard signal transmission with chaos;⁷ (b) standard scheme combined with the channel equalization method based on complimentary filtering in the feedback loop of the response system; (c) injecting the information signal filtered with a filter matching the channel filter, restores the symmetry between encoder and decoder

subtracting the filtered feedback signal x_d from the received signal $u_d \equiv c(t) \otimes (x_e(t) + m(t))$ ($c(t)$ is the impulse response function of the channel, and \otimes denotes linear convolution). Unfortunately, when channel distortions are present, recovered signal remains corrupted because the symmetry between the encoder and decoder is broken. The decoder receives a sum of the filtered modulated chaotic signal with the *filtered* message, whereas the encoder is fed by the sum of the same modulated chaotic signal with the *unfiltered* message. We illustrate this asymmetry by a numerical example of transmission of a sine wave $m(t) = 0.1 \sin(\omega t)$ from the driving to the response system using the following three-dimensional chaotic systems²⁰

$$\frac{dx_e}{dt} = y_e, \quad \frac{dy_e}{dt} = z_e, \quad \frac{dz_e}{dt} = -az_e - by_e - cx_e + dG(u_e), \quad u_e = x_e + m(t) \tag{1}$$

where

$$G(u) = \begin{cases} 1 - u^2 & \text{at } -0.95 < u < 2 \\ 0.0025 & \text{at } u < -0.95 \\ -3 & \text{at } u > 2 \end{cases} \tag{2}$$

and parameters are $a = 0.73, b = 2.63, c = 0.2, d = 2.0$. The dynamics of the decoder is described by the similar equations where subscript ‘e’ is replaced by ‘d’, and $u_d(t)$ is the signal $u_e(t)$ transmitted through a channel, $u_d(t) = c(t) \otimes u_e(t)$. Here $c(t) = \Omega_c^{-1} \exp(-\Omega_c t)$ with a cutoff frequency $\Omega_c = 5$ rad/s. The information is recovered by the subtraction $m_r(t) = u_d(t) - x_d(t)$. In Figure 3 the r.m.s. recovery error relative to the standard deviation of the information signal is shown as a function of the signal frequency ω . For small ω , the filter does not affect the information signal and so the quality of recovery is high. However, for higher frequencies (which are still within the passband of the channel) the filter starts to affect the amplitude as well as the phase of the transmitted information signal and recovery error grows rapidly. The large peak near 1.6 rad/s is probably due to a resonance between the sinusoidal signal and the chaotic oscillator.

The only possible way to provide an exact synchronization when the information component is present, is to learn the channel transfer function *at the transmitter* and, using a proper filter, pre-process the information signal accordingly before injecting it into a feedback loop of the encoder. The block-diagram of this

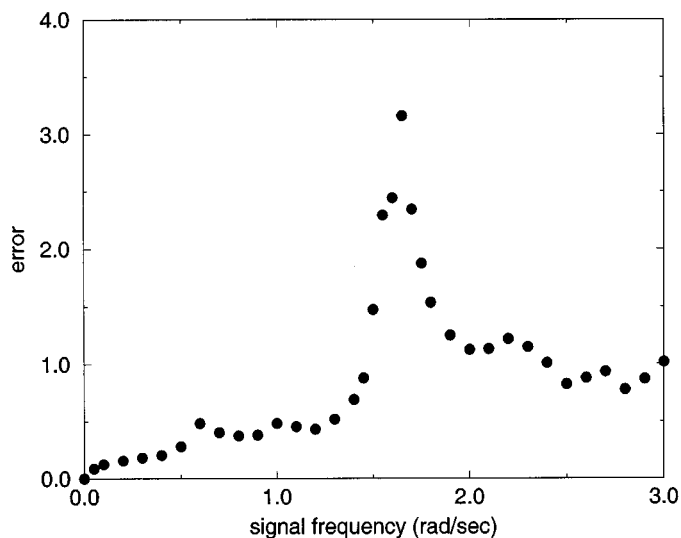


Figure 3. Relative r.m.s. signal recovery error $\|m_r - m\|$ for sine wave $m(t) = 0.1 \sin(\omega t)$ as a function of its frequency ω for information transmission using system (1)–(2) and the method of Figure 2(a)

modification is shown in Figure 2(c). The decoding procedure which is the same as in Figure 2(b), yields a *filtered* message $c(t) \otimes m(t)$ or exactly the same signal as one could possibly receive without mixing it with a chaotic component. Numerical simulations show that indeed the recovery error remains within the accuracy of the integrator (10^{-10} for signal frequencies throughout the passband of the channel). Thus, pre-processing of the information signal cures the problem of asymmetry in the encoder–decoder and yields exact synchronization of chaos and signal recovery. However, as was noted before, this method may cause the instability which may occur due to the compensating signal in the feedback loop of the response system.

3. COMMUNICATION WITH CHAOTIC SIGNALS—NARROW-BAND CHAOTIC SYSTEM DESIGN

In this section we consider a different approach to achieve synchronization over band-limited channel by preparing a narrow-band chaotic signal which would not be affected by the channel distortions. If, in addition, the information message is also narrow-band, it can be recovered at the decoder without distortion.

Design of chaotic waveforms generators usually relies upon the utilization of known models in form of differential equations, which are capable of producing chaotic dynamics. In most of the cases these models do not have parameters which can be directly used to shape the power spectra of output signals in a systematic manner. Therefore, it is desirable to develop means for modifying chaos oscillators in order to introduce such parameters. Clearly, such modification can be achieved using additional filters in the design, however, it is important to study the implications of such modification. Here, we consider two methods of creating analog chaotic encoders/decoders which synchronize via narrow-band chaotic signals.

3.1. Method 1

The first method is shown schematically in Figure 4(a). It is somewhat similar to the channel compensation method of Figure 1(b), however, the filters here are independent of the channel characteristics. The x -component of the encoder is divided into a filtered part $x_e^\phi = \phi \otimes x_e$ and a complimentary part $\bar{x}_e^\phi = x_e - x_e^\phi$. The information signal $m(t)$ is added to x_e^ϕ and then the sum $u_e = x_e^\phi + m(t)$ is sent to the decoder. This sum is narrow-band since by assumption both the message and the filter ϕ are narrow-band. Further, the complimentary signal \bar{x}_e^ϕ is added to u_d . Therefore, the signal which is injected into the non-linear element of the system is simply $x_e + m$, as if there were no filter in the feedback loop. At the decoder, the x_d -component is similarly split into the filtered part $x_d^\phi = \phi \otimes x_d$ and its complimentary part $\bar{x}_d^\phi = x_d - x_d^\phi$. Component \bar{x}_d^ϕ is added to the received signal u_d and the sum is injected into the non-linear element. The information signal is recovered as a difference $m_r(t) = u_d - x_d^\phi$. It is easy to see that if the transmitted signal is passed through a channel without distortion ($u_d = u_e$), the message is recovered at the decoder exactly. We call this scheme narrow-band, closed-loop design (NBCL).

One advantage of this scheme is that no matter how narrow the filter ϕ is, the chaotic dynamics of the systems remains unchanged. Yet the transmitted chaotic signal can be made arbitrarily narrow. The disadvantage, however, is that this method again does not guarantee stable synchronization, since the component \bar{x}_d^ϕ provides a closed feedback loop in the response system. The narrower the filter ϕ is, the more power is passed through \bar{x}_d^ϕ and the synchronization becomes less stable. At some level of spectrum reduction the response system loses synchronization and de-synchronization bursts lead to large errors in the recovered information signal. Hence, the interplay of two counteracting factors, chaos distortions in the channel and stability of synchronization, determines the quality of transmission in this scheme.

We applied this method to the chaotic encoder and decoder described by equation (1). In the feedback loops of the oscillators we used a fourth-order low-pass elliptic filter with a stop-band attenuation of 10 dB, passband gain of 0.6 and the cutoff frequency Ω_c which was adjusted in order to optimize the performance of the transmission for a given channel. Figure 5 compares the r.m.s. error for transmission of the sine wave $0.1 \sin(\omega t)$ with $\omega = 0.5$ rad/s with the case when no filtration is applied to the transmitted chaotic signal. For

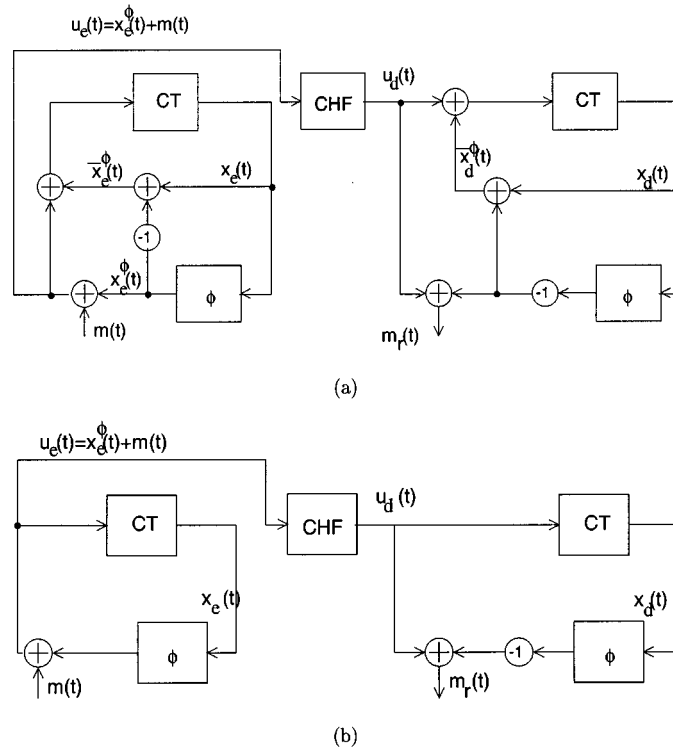


Figure 4. (a) Block-diagram of information transmission with the narrow-band chaotic masking signal based on complimentary filtering in the closed feedback loops of both driving and response systems (NBCL); (b) same for the open-loop response system (NBOL)

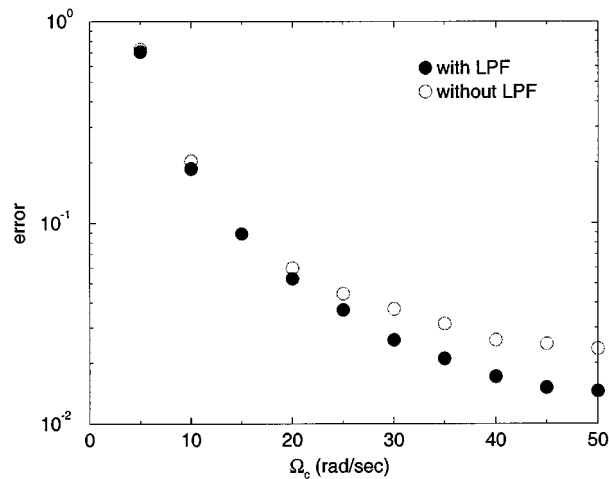
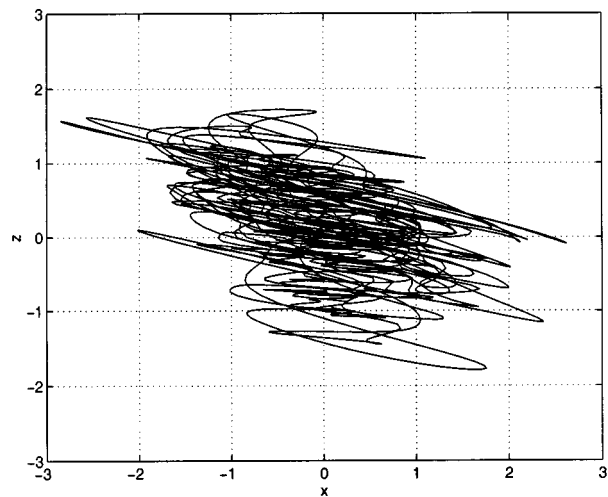


Figure 5. (a) Relative r.m.s. signal recovery error for sine wave $m(t) = 0.1 \sin(\omega t)$ as a function of the cutoff frequency of the channel for the NBCL scheme Figure 4(a) with and without LP filter in the feedback loop

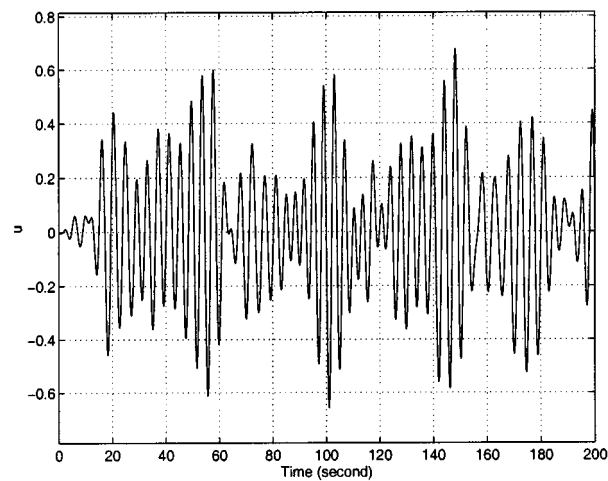
relatively high cutoff frequencies of the channel Ω_c pre-filtration yields certain improvements which however vanishes for lower Ω_c . The reason for this is that it is impossible to reduce the cutoff frequency of the LP filters below some value as the synchronization becomes unstable for low Ω_ϕ .

3.2. Method 2

As we mentioned above, Method 1 (NBCL) guarantees the preserving of chaotic behaviour of both driving and response systems for any filters used in their feedback loops, but it does not guarantee the preserving of self-synchronization between encoder and decoder. It is desirable to couple two chaotic system by a narrow-band signal and still have a robust chaotic synchronization. This goal can be achieved by changing the structure of chaotic oscillators in such a way that one of its internal variables is itself narrow-band, and therefore the decoder can be made open-loop and so unconditionally stable. We call this design narrow-band, open-loop (NBOL).



(a)



(b)

Figure 6. (a) Chaotic attractor of the system (1) with non-linearity $G(u) = \sin(15u)$ and fourth-order Butterworth bandpass filter (center frequency 1.5 rad/s, bandwidth 0.5 rad/s) in the feedback loop. (b) Time series of the output $u(t)$

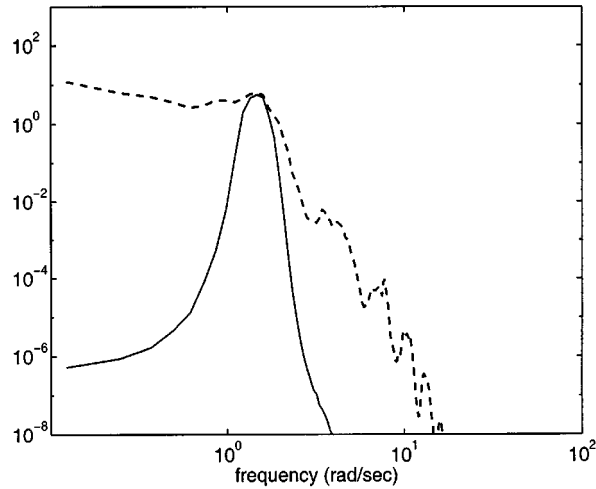


Figure 7. Power spectra of the narrow-band signal $u(t)$ from Figure 6(b) (solid line) and corresponding signal $x(t)$ before the filter (dashed line)

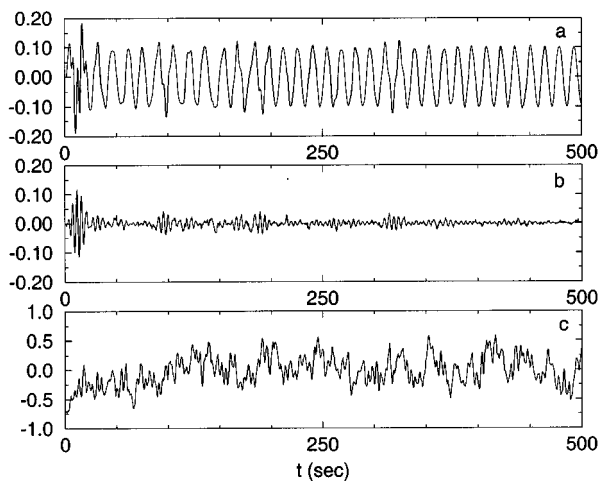


Figure 8. Recovered signal $m_r(t)$: (a) and recovery error $m_r(t) - m(t)$; (b) for the system Figure 4(b) with chaos generator described in Figure 6; and (c) recovered signal $m_r(t)$ for the same system with no bandpass filters in feedback loops of chaos generators

We introduce a narrow bandpass filter in the feedback loops of both oscillators but eliminate compensating feedback loops (Figure 4(b)). In equation (1) this method corresponds to choosing $u_e \equiv \phi \otimes x_e + m(t)$. Such filtering however affects the dynamics of the systems and can, in principle, destroy its chaotic attractor. In order to maintain chaos, one needs to have strong non-linearity (function $G(u)$) in the system, so the narrow-band signal u_e generates many spectral components which in turn interact with other system variables. In numerical simulations, we chose $G(u) = \sin(Au)$, and parameter A characterizes the strength of non-linearity (we used $A = 15$). In the feedback loops fourth-order Butterworth bandpass filters are placed (centre frequency 1.5 rad/s, bandwidth $\Delta\omega = 0.5$ rad/s). The dynamics of this system remains chaotic (see the attractor in (x, z) projection in Figure 6(a)), but the variable u at the output of the filter is indeed narrow-band (see Figures 6(b) and 7). We coupled two such chaotic systems via a channel which is simulated by the

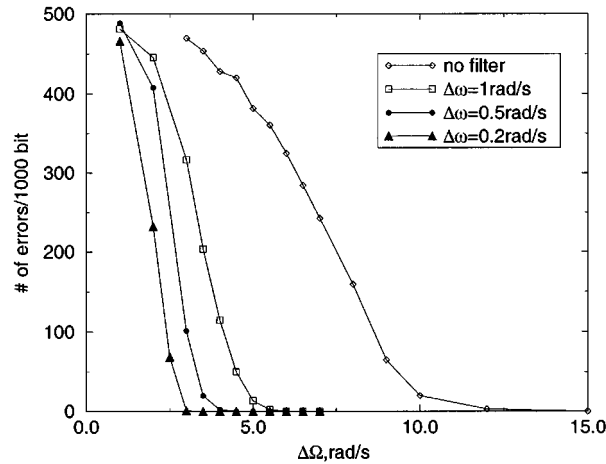


Figure 9. Error rate (per 1000 bits) for binary transmission through the low-pass channel filter for various bandwidths of BPFs in the encoder and decoder

low-pass sixth-order Chebyshev type II filter with cutoff frequency $\Omega_c = 5$ rad/s and 40 dB attenuation in the stop-band. We transmitted two types of signals using this system. First, a sinusoidal signal $m(t) = 0.1 \sin(1.5t)$ which frequency coincides with the centre frequency of feedback filters, was injected in the encoder. The restored signal and the recovery error are shown in Figure 8(a) and 8(b). One can see that the signal recovery is indeed very good (signal-to-noise ratio is about 14 dB). For comparison, we show in Figure 8(c) the recovered signal for this system without bandpass filters in the feedback loops. The 'recovered' signal is not even reminiscent of the expected sine wave.

We also carried a number of tests involving transmission of a pseudo-random binary sequence s_n of 1 and -1 using a pulse-band modulation

$$m(t) = 0.1 \sum_{n=0}^N s_n g(t - nT) \quad (3)$$

for various filters in the feedback loops and in the channel. Function $g(t) = \sin(1.5t)/(1.5t)$ was chosen to minimize inter-symbol interference at the demodulating stage, and $T = \pi/1.5$ s. After demodulating this signal we computed error rate per 1000 bits for a number of values of Ω and $\Delta\omega$. This data is presented in Figure 9. For comparison, we also computed the error rate for the case when there is no filter in the feedback loops of the systems. It is easy to see that the using filters in the systems allows to reduce drastically the error rate once the signal u fits into the channel passband.

4. EXPERIMENTAL RESULTS

As a generator of chaos we used an analog non-linear circuit which consists of serial connection of two input-output elements, non-linear converter (NLC), see Figure 10, and a linear feedback (LFB), see Figure 11. More details about implementation this chaotic circuit can be found in Reference 21. Being connected in a loop, NLC and LFB give rise to self-sustained chaotic oscillations. To design chaotic encoder and decoder on the basis of this chaos generators we use the method described in Reference 7 (see Figure 2(a)).

To model limited bandwidth of the communication channel we connected the encoder and decoder through a second-order low-pass voltage-controlled voltage-source (VCVS) active filter. Parameters of the

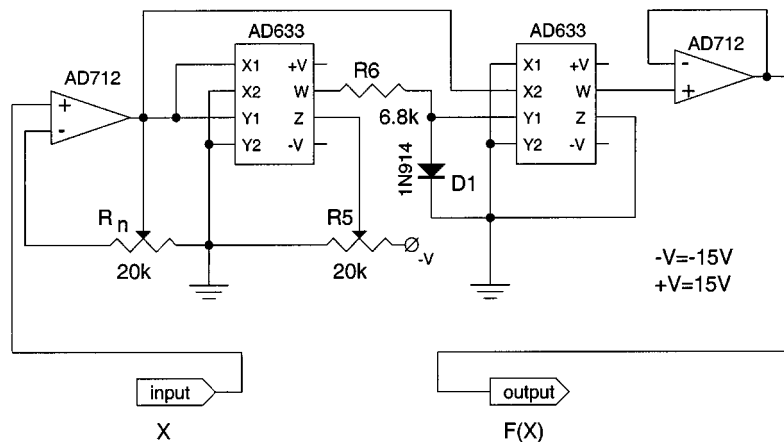


Figure 10. Diagram of the nonlinear converter (NLC)

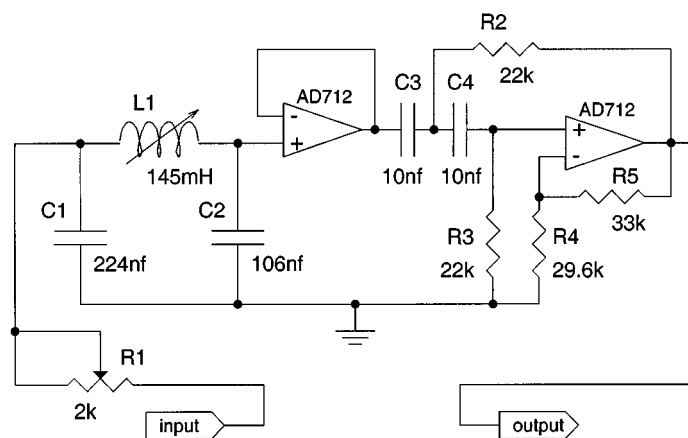


Figure 11. Diagram of the linear feedback element (LFB)

filter were selected to make a Bessel filter.²² As a control parameter of the filter we use its cutoff frequency f_c measured at -3 dB.

In the experiments we compared two schemes of chaos communication. The first is the original scheme implemented with open-loop chaotic decoder, see Figure 2(a). The second one is the modified scheme Figure 4(b) that utilizes low-pass filters (LPF) to design the narrow bandwidth signal in the feedback of the chaos generator. Parameters of these filters were selected to make Chebyshev filter with the cutoff frequency $f_\phi = 300$ Hz measured at -2 dB. Chaotic attractors generated by the chaotic encoders of the original and the modified schemes with $m(t) = 0$ are shown in Figures 12(a) and 12(b), respectively.

In the experiment we evaluated the quality of synchronization as a function of the cutoff frequency of filter CHF. For this purpose, we chose simply $m(t) = 0$, and measured $u(t)$ and $m_r(t)$. Using these waveforms sampled at 10 kHz we calculated maximal deviation and standard deviation of $m_r(t)$ normalized with respect to the magnitude of the transmitted signal (Figure 13). The plots demonstrate that using additional filters in the encoder and decoder can significantly reduce errors caused by degrading the chaotic signal in the limited bandwidth channel. In the case of NBOL errors becomes significant only when the channel has the cutoff

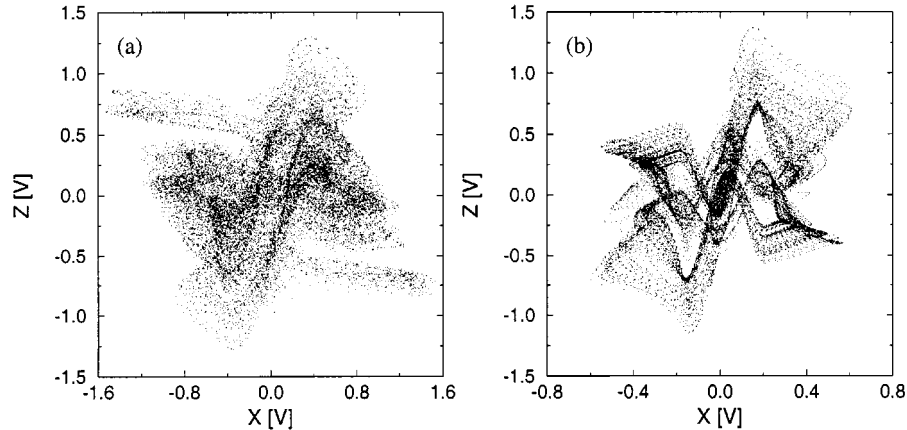


Figure 12. (X, Z) -projections of chaotic attractors generated by the encoder with $m(t) = 0$. $X(t)$ is the input of NLC, see Figure 10. $Z(t)$ is voltage measured across capacitor C_1 in LFB, see Figure 11. (a) Original encoder, $R_n = 1103 \Omega$. (b) Modified narrow-band encoder, $R_n = 446 \Omega$. Resistor R_n controls the gain in the NLC, see Figure 10

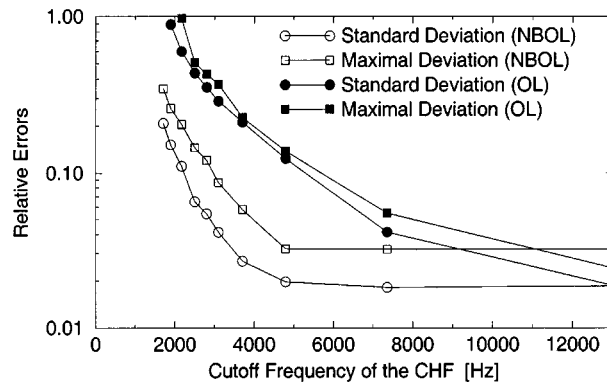


Figure 13. Relative errors vs. cutoff frequency of the filter in the communication link in the experiment with electronic circuits

frequency less than 4 kHz. For the broader channel the errors are due to the slight mismatch between the parameters of the encoder and decoder.

To demonstrate the feasibility of the communication with NBOL through the narrow bandwidth channel we used voice signal as $m(t)$ and measured $m(t)$, $u(t)$ and $m_r(t)$ simultaneously. Figure 14 shows the results of the measurements obtained for the CHF with $f_c = 4.8$ kHz. Although one can notice imperfections of the recovered signal, the quality of the communication is, in fact, pretty good. Note that for this cutoff frequency of the channel filter the standard scheme (without LPF) does not yield any meaningful signal recovery (cf. Figure 13). To quantify the quality of communication with this settings we measured signal-to-noise ratio, which is in our case is signal-to-chaos ratio, for the sinusoidal signal $m(t)$ with frequency 300 Hz and amplitude 80 mV. This amplitude was chosen to be close to the maximum amplitude of audio signal shown in Figure 14. In the communication channel SNR was -13.5 dB, and in the recovered signal SNR was 10.3 dB.

5. CONCLUSIONS

In this work we have explored various possibilities of transmitting analog information, which is scrambled by chaos, through a band-limited channel. Two main classes of system design are considered. First class corrects

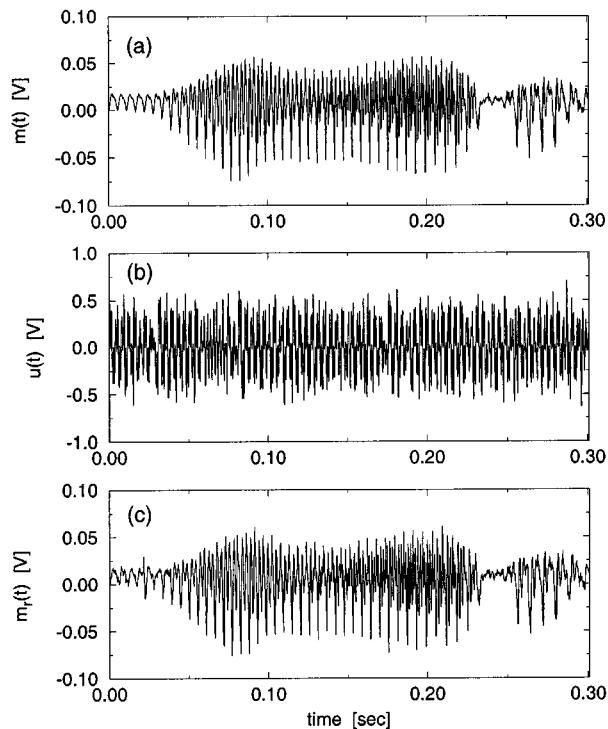


Figure 14. Waveforms of the original information signal (a), the encoded signal measured in the communication link (b), and the recovered information signal (c). The cutoff frequency of the CHF 4.8 kHz, sampling rate 30000 samples per second

the channel distortions either by using inverse filter at the receiver, or by augmenting the transmitted signal by the chaotic signal generated by the decoding chaotic system passed through a complimentary filter. Utilization of these approaches at significant levels of channel distortions can be limited by the instabilities of the synchronization regime which would result in the poor signal recovery.

The second class is based on the modification of the structure of the chaotic signal *before* sending it through the channel, so it is by construction made suitable for undistorted transmission. This approach does not require learning the channel characteristic and can provide robust signal transmission. If combined with open-loop design of the response system (Method 2 of Section 3), it guarantees the robust stable synchronization of chaos. We demonstrated in numerical simulations and experiments with electronic circuits that the quality of signal recovery improves greatly as compared to transmission with wide-spectrum chaotic signals.

ACKNOWLEDGEMENTS

We are grateful to H. Abarbanel, L. Larsen, M. M. Sushchik, and A. R. Volkovskii for helpful discussions. This work was supported by the U.S. Department of Energy (grant DE-FG03-95ER14516) and the U.S. Army Research Office (MURI grant DAAG55-98-1-0269).

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